# Lower Tail Test of Population Mean with Known Variance

The null hypothesis of the **lower tail test of the population mean**can be expressed as follows:

![μ ≥ μ0
]()

where *μ*0 is a hypothesized lower bound of the true population mean *μ*.

Let us define the test statistic *z*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [population standard deviation](http://www.r-tutor.com/node/43) *σ*:

z = ¯x−√μ0-
    σ∕  n


Then the null hypothesis of the lower tail test is to be *rejected*if *z*≤−*zα* , where *zα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

#### Problem

Suppose the manufacturer claims that the mean lifetime of a light bulb is more than 10,000 hours. In a sample of 30 light bulbs, it was found that they only last 9,900 hours on average. Assume the population standard deviation is 120 hours. At .05 significance level, can we reject the claim by the manufacturer?

#### Solution

The null hypothesis is that *μ*≥ 10000. We begin with computing the test statistic.

> xbar = 9900            # sample mean   
> mu0 = 10000            # hypothesized value   
> sigma = 120            # population standard deviation   
> n = 30                 # sample size   
> z = (xbar−mu0)/(sigma/sqrt(n))   
> z                      # test statistic   
[1] −4.5644

We then compute the critical value at .05 significance level.

> alpha = .05   
> z.alpha = qnorm(1−alpha)   
> −z.alpha               # critical value   
[1] −1.6449

#### Answer

The test statistic -4.5644 is less than the critical value of -1.6449. Hence, at .05 significance level, we reject the claim that mean lifetime of a light bulb is above 10,000 hours.

#### Alternative Solution

Instead of using the critical value, we apply the pnorm function to compute the lower tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≥ 10000.

> pval = pnorm(z)   
> pval                   # lower tail p−value   
[1] 2.5052e−06

# Upper Tail Test of Population Mean with Known Variance

The null hypothesis of the **upper tail test of the population mean**can be expressed as follows:

![μ ≤ μ0
]()

where *μ*0 is a hypothesized upper bound of the true population mean *μ*.

Let us define the test statistic *z*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [population standard deviation](http://www.r-tutor.com/node/43) *σ*:

    ¯x− μ0
z = σ∕√n--


Then the null hypothesis of the upper tail test is to be *rejected*if *z*≥ *zα* , where *zα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

#### Problem

Suppose the food label on a cookie bag states that there is at most 2 grams of saturated fat in a single cookie. In a sample of 35 cookies, it is found that the mean amount of saturated fat per cookie is 2.1 grams. Assume that the population standard deviation is 0.25 grams. At .05 significance level, can we reject the claim on food label?

#### Solution

The null hypothesis is that *μ*≤ 2. We begin with computing the test statistic.

> xbar = 2.1             # sample mean   
> mu0 = 2                # hypothesized value   
> sigma = 0.25           # population standard deviation   
> n = 35                 # sample size   
> z = (xbar−mu0)/(sigma/sqrt(n))   
> z                      # test statistic   
[1] 2.3664

We then compute the critical value at .05 significance level.

> alpha = .05   
> z.alpha = qnorm(1−alpha)   
> z.alpha                # critical value   
[1] 1.6449

#### Answer

The test statistic 2.3664 is greater than the critical value of 1.6449. Hence, at .05 significance level, we reject the claim that there is at most 2 grams of saturated fat in a cookie.

#### Alternative Solution

Instead of using the critical value, we apply the pnorm function to compute the upper tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≤ 2.

> pval = pnorm(z, lower.tail=FALSE)   
> pval                   # upper tail p−value   
[1] 0.0089802

# Two-Tailed Test of Population Mean with Known Variance

The null hypothesis of the **two-tailed test of the population mean**can be expressed as follows:

![μ = μ0
]()

where *μ*0 is a hypothesized value of the true population mean *μ*.

Let us define the test statistic *z*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [population standard deviation](http://www.r-tutor.com/node/43) *σ*:

    ¯x− μ0
z = σ∕√n--


Then the null hypothesis of the two-tailed test is to be *rejected*if *z*≤−*zα∕*2 or *z*≥ *zα∕*2 , where *zα∕*2 is the 100(1 − *α∕*2) [percentile](http://www.r-tutor.com/node/38) of the [standard normal distribution](http://www.r-tutor.com/node/58).

#### Problem

Suppose the mean weight of King Penguins found in an Antarctic colony last year was 15.4 kg. In a sample of 35 penguins same time this year in the same colony, the mean penguin weight is 14.6 kg. Assume the population standard deviation is 2.5 kg. At .05 significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

#### Solution

The null hypothesis is that *μ*= 15*.*4. We begin with computing the test statistic.

> xbar = 14.6            # sample mean   
> mu0 = 15.4             # hypothesized value   
> sigma = 2.5            # population standard deviation   
> n = 35                 # sample size   
> z = (xbar−mu0)/(sigma/sqrt(n))   
> z                      # test statistic   
[1] −1.8931

We then compute the critical values at .05 significance level.

> alpha = .05   
> z.half.alpha = qnorm(1−alpha/2)   
> c(−z.half.alpha, z.half.alpha)   
[1] −1.9600  1.9600

#### Answer

The test statistic -1.8931 lies between the critical values -1.9600 and 1.9600. Hence, at .05 significance level, we do *not*reject the null hypothesis that the mean penguin weight does not differ from last year.

#### Alternative Solution

Instead of using the critical value, we apply the pnorm function to compute the two-tailed **p-value**of the test statistic. It doubles the *lower*tail p-value as the sample mean is *less*than the hypothesized value. Since it turns out to be greater than the .05 significance level, we do *not*reject the null hypothesis that *μ*= 15*.*4.

> pval = 2 ∗ pnorm(z)    # lower tail   
> pval                   # two−tailed p−value   
[1] 0.058339

# Upper Tail Test of Population Mean with Unknown Variance

The null hypothesis of the **upper tail test of the population mean**can be expressed as follows:

![μ ≤ μ0
]()

where *μ*0 is a hypothesized upper bound of the true population mean *μ*.

Let us define the test statistic *t*in terms of the [sample mean](http://www.r-tutor.com/node/35), the sample size and the [sample standard deviation](http://www.r-tutor.com/node/43) *s*:

    ¯x− μ0
t = s∕√n--


Then the null hypothesis of the upper tail test is to be *rejected*if *t*≥ *tα* , where *tα* is the 100(1 − *α*) [percentile](http://www.r-tutor.com/node/38) of the [Student t distribution](http://www.r-tutor.com/node/59) with *n*− 1 degrees of freedom.

#### Problem

Suppose the food label on a cookie bag states that there is at most 2 grams of saturated fat in a single cookie. In a sample of 35 cookies, it is found that the mean amount of saturated fat per cookie is 2.1 grams. Assume that the sample standard deviation is 0.3 gram. At .05 significance level, can we reject the claim on food label?

#### Solution

The null hypothesis is that *μ*≤ 2. We begin with computing the test statistic.

> xbar = 2.1             # sample mean   
> mu0 = 2                # hypothesized value   
> s = 0.3                # sample standard deviation   
> n = 35                 # sample size   
> t = (xbar−mu0)/(s/sqrt(n))   
> t                      # test statistic   
[1] 1.9720

We then compute the critical value at .05 significance level.

> alpha = .05   
> t.alpha = qt(1−alpha, df=n−1)   
> t.alpha                # critical value   
[1] 1.6991

#### Answer

The test statistic 1.9720 is greater than the critical value of 1.6991. Hence, at .05 significance level, we can reject the claim that there is at most 2 grams of saturated fat in a cookie.

#### Alternative Solution

Instead of using the critical value, we apply the pt function to compute the upper tail **p-value**of the test statistic. As it turns out to be less than the .05 significance level, we reject the null hypothesis that *μ*≤ 2.

> pval = pt(t, df=n−1, lower.tail=FALSE)   
> pval                   # upper tail p−value   
[1] 0.028393